



Esercizio Multiplazione

- ❑ Si consideri un canale di velocità $C=2,048$ Mbit/s
- ❑ si vogliano ricavare canali di velocità $c=64$ kbit/s con un ritardo di adattamento massimo di 10 ms
- ❑ si calcoli:
 - il numero di canali ottenibili
 - la durata massima di slot
 - il numero di bit per slot

$$N = C/c = 2048/64 = 32$$

$$T_a = n_i / c = 10 \text{ ms}$$

$$n_i = T_a c = 0,01 \cdot 64000 = 640$$

$$T_i = n_i / C = 0,3125 \text{ ms}$$

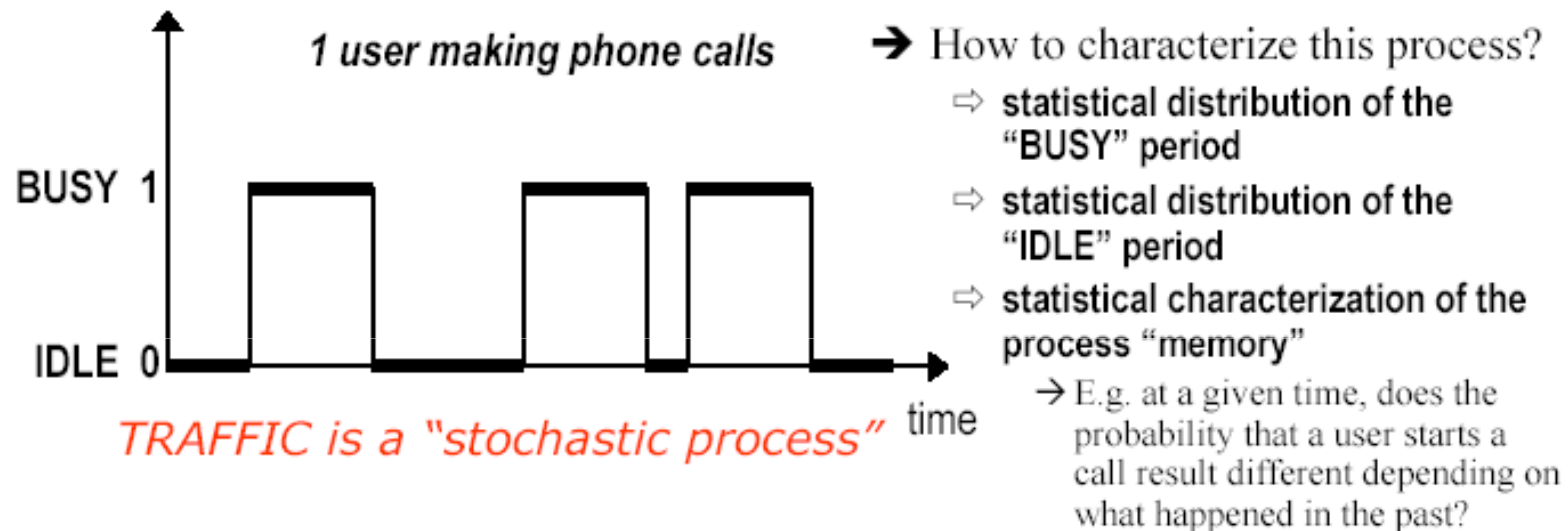


Cellular Coverage Concepts

Lecture 2.4 teletraffic considerations, teletraffic planning



Traffic generated by one user (statistical notion of traffic)



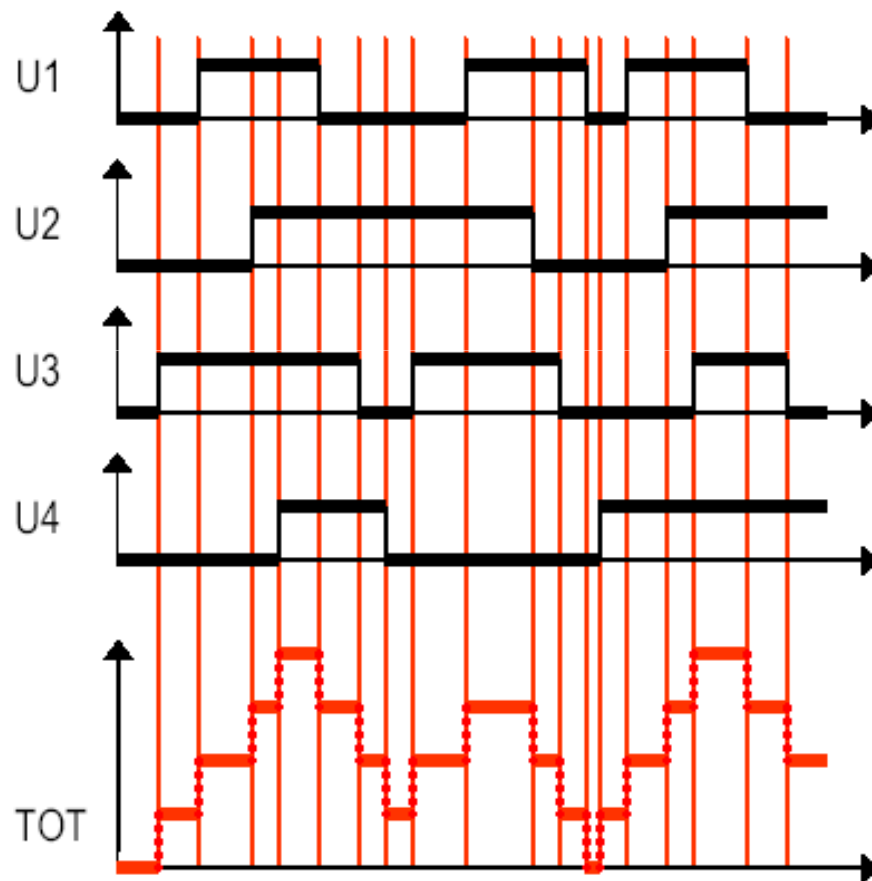
→ Traffic characterization suitable for traffic engineering

$$\begin{aligned}\text{traffic intensity } A_i &= \lim_{\Delta t \rightarrow \infty} \frac{\text{amount of busy time in } \Delta t}{\Delta t} = \\ &= (\text{average number } \lambda \text{ of calls per hour}) \times (\text{average call duration } \tau) = \\ &= \text{probability that, at a random time } t, \text{ user is in BUSY state} = \\ &= \text{mean process value}\end{aligned}$$

All equivalent (if stationary process)



Traffic generated by more than one users



Traffic intensity
(adimensional, measured in Erlangs):

$$A = \sum_{i=1}^4 A_i = 4 A_i$$

$$P[k \text{ active calls}] = \binom{4}{k} A_i^k (1 - A_i)^{4-k}$$

$$E[\text{active calls}] = 4 \cdot A_i = A$$



example

- 5 users
- Each user makes an average of 3 calls per hour
- Each call, in average, lasts for 4 minutes

$$A_i = 3 \left[\frac{\text{calls}}{\text{hour}} \right] \times \frac{4}{60} [\text{hours}] = \frac{1}{5} [\text{erl}]$$

$$A = 5 \times \frac{1}{5} [\text{erl}] = 1 [\text{erl}]$$

Meaning: in average, there is 1 active call;
but the actual number of active calls varies
from 0 (no active user) to 5 (all users active),
with given probability

number of active users	probability
0	0,327680
1	0,409600
2	0,204800
3	0,051200
4	0,006400
5	0,000320



Second example


- 30 users
- Each user makes an average of 1 calls per hour
- Each call, in average, lasts for 4 minutes

$$A = 30 \times \left(1 \cdot \frac{4}{60} \right) = 2 \text{ Erlangs}$$

SOME NOTES:

- In average, 2 active calls (intensity A);
- Frequently, we find up to 4 or 5 calls;
- Prob(n.calls>8) = 0.01%
- More than 11 calls only once over 1M

TRAFFIC ENGINEERING: how many channels to reserve for these users!



n. active users	binom	probab	cumulat
0	1	1,3E-01	0,126213
1	30	2,7E-01	0,396669
2	435	2,8E-01	0,676784
3	4060	1,9E-01	0,863527
4	27405	9,0E-02	0,953564
5	142506	3,3E-02	0,987006
6	593775	1,0E-02	0,996960
7	2035800	2,4E-03	0,999397
8	5852925	5,0E-04	0,999898
9	14307150	8,7E-05	0,999985
10	30045015	1,3E-05	0,999998
11	54627300	1,7E-06	1,000000
12	86493225	1,9E-07	1,000000
13	119759850	1,9E-08	1,000000
14	145422675	1,7E-09	1,000000
15	155117520	1,3E-10	1,000000
16	145422675	8,4E-12	1,000000
17	119759850	5,0E-13	1,000000
18	86493225	2,6E-14	1,000000
19	54627300	1,2E-15	1,000000
20	30045015	4,5E-17	1,000000
21	14307150	1,5E-18	1,000000
22	5852925	4,5E-20	1,000000
23	2035800	1,1E-21	1,000000
24	593775	2,3E-23	1,000000
25	142506	4,0E-25	1,000000
26	27405	5,5E-27	1,000000
27	4060	5,8E-29	1,000000
28	435	4,4E-31	1,000000
29	30	2,2E-33	1,000000
30	1	5,2E-36	1,000000



A note on binomial coefficient computation

$$\binom{60}{12} = \frac{60!}{12!48!} = 1.39936e+12$$

but $60! = 8.32099e+81$ (*overflow problems!!*)

$$\begin{aligned}\binom{60}{12} &= \exp\left(\log\left(\binom{60}{12}\right)\right) = \exp(\log(60!) - \log(12!) - \log(48!)) = \\ &= \exp\left(\sum_{i=1}^{60} \log(i) - \sum_{i=1}^{12} \log(i) - \sum_{i=1}^{48} \log(i)\right) \quad (\text{no overflow!! before exp...})\end{aligned}$$

$$\begin{aligned}\binom{60}{12} A_i^{12} (1 - A_i)^{48} &= \\ &= \exp\left(\sum_{i=1}^{60} \log(i) - \sum_{i=1}^{12} \log(i) - \sum_{i=1}^{48} \log(i) + 12 \log(A_i) + 48 \log(1 - A_i)\right) \\ &\quad (\text{no overflow!! never!})\end{aligned}$$



Infinite Users

Assume M users, generating an overall traffic intensity A
(i.e. each user generates traffic at intensity $A_i = A/M$).

We have just found that

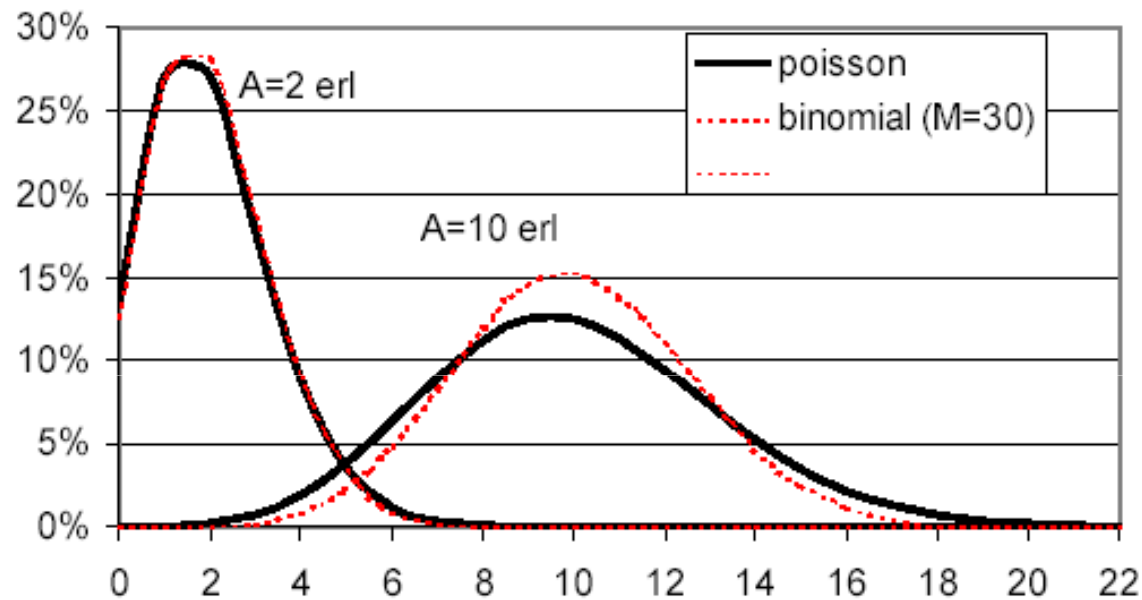
$$P[k \text{ active calls}, M \text{ users}] = \binom{M}{k} A_i^k (1 - A_i)^{M-k} = \frac{M!}{(M-k)!k!} \left(\frac{A}{M}\right)^k \frac{\left(1 - \frac{A}{M}\right)^M}{\left(1 - \frac{A}{M}\right)^k}$$

Let $M \rightarrow \infty$, while maintaining the same overall traffic intensity A

$$\begin{aligned} P[k \text{ active calls}, \infty \text{ users}] &= \lim_{M \rightarrow \infty} \frac{M!}{(M-k)!} \cdot \frac{1}{k!} \cdot \frac{A^k}{M^k} \cdot \left(1 - \frac{A}{M}\right)^M \cdot \left(1 - \frac{A}{M}\right)^{-k} = \\ &= \frac{A^k}{k!} \cdot \lim_{M \rightarrow \infty} \frac{M(M-1) \cdots (M-k+1)}{M^k} \cdot \left[\left(1 - \frac{A}{M}\right)^{-\frac{M}{A}} \right]^{-A} \cdot \left(1 - \frac{A}{M}\right)^{-k} = e^{-A} \frac{A^k}{k!} \end{aligned}$$



Poisson Distribution



$$P_k(A) = e^{-A} \frac{A^k}{k!}$$

Very good matching with Binomial
(when M large with respect to A)

Much simpler to use than Binomial
(no annoying queueing theory complications)



Limited number of channels

THE most important problem
in circuit switching

→ The number of channels
 C is less than the
number of users M
(eventually infinite)

→ Some offered calls will
be "blocked"

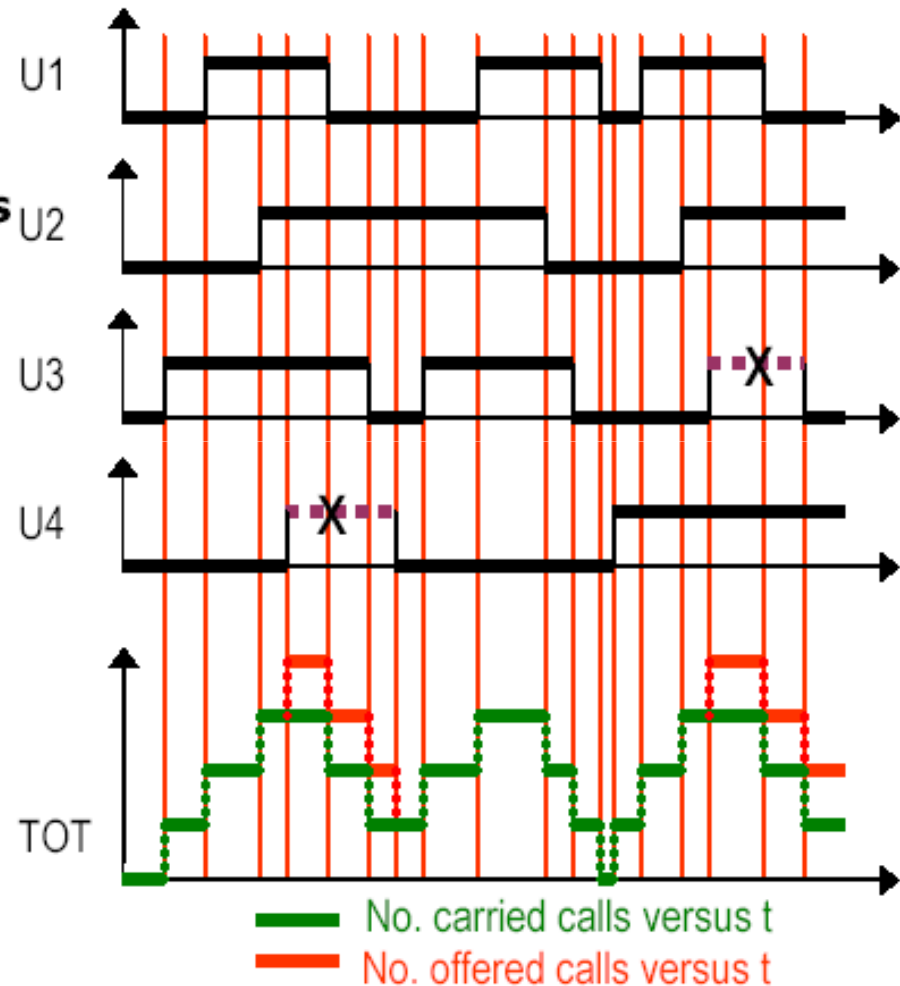
→ What is the blocking
probability?

⇒ We have an expression for
 $P[k \text{ offered calls}]$

⇒ We must find an expression for
 $P[k \text{ accepted calls}]$

⇒ As:

$$P[\text{block}] = P[C \text{ accepted calls}]$$





Channel utilization probability

→ **C channels available**

→ **Assumptions:**

⇒ Poisson distribution (infin. users)

⇒ Blocked calls cleared

→ **It can be proven (from Markov Processes theory) that:**

$$P[k \text{ calls in the system}] =$$

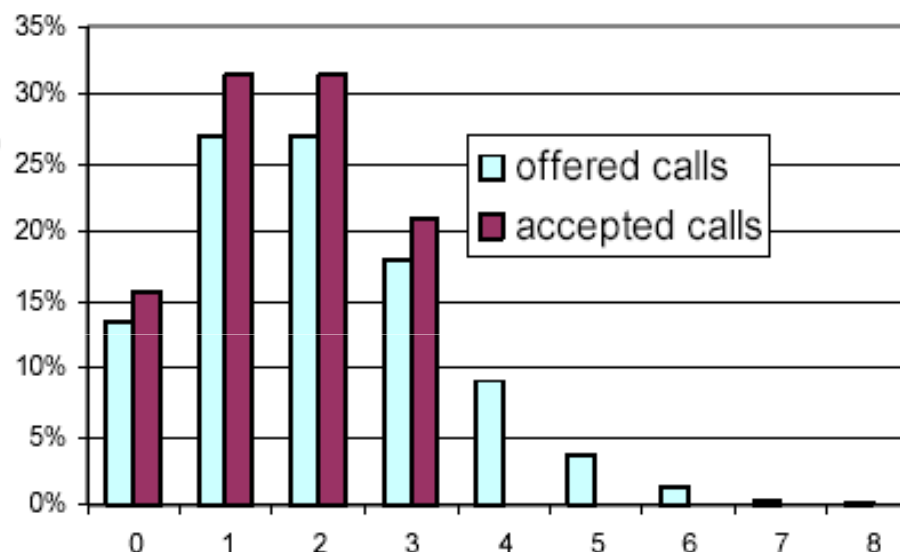
$$= \frac{P[k \text{ offered calls}]}{\sum_{i=0}^C P[i \text{ offered calls}]}$$

(very simple result!)

→ **Hence:**

$$P[\text{system full}] = P[C \text{ accepted calls}] = \frac{P[C \text{ offered calls}]}{\sum_{i=0}^N P[i \text{ offered calls}]}$$

offered traffic: 2 erl - C=3





Blocking probability: Erlang-B

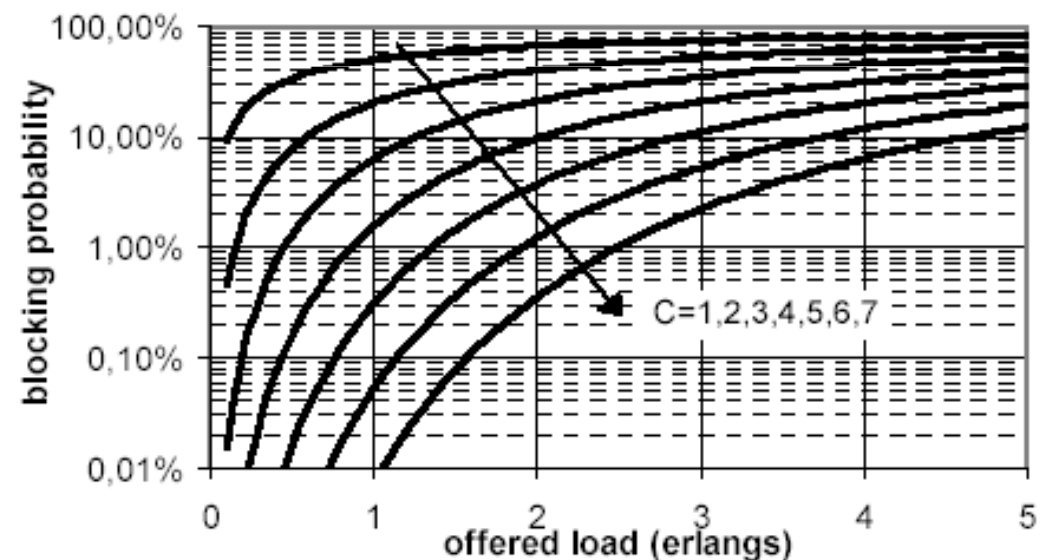
→ **Fundamental formula for telephone networks planning**

⇒ A_o = offered traffic in Erlangs

$$\Pi_{block} = \frac{\frac{A_o^C}{C!}}{\sum_{j=0}^C \frac{A_o^j}{j!}} = E_{1,C}(A_o)$$

→ **Efficient recursive computation available**

$$E_{1,C}(A_o) = \frac{A_o E_{1,C-1}(A_o)}{C + A_o E_{1,C-1}(A_o)}$$





NOTE: finite users

- Erlang-B obtained for the infinite users case
- It is easy (from queueing theory) to obtain an explicit blocking formula for the finite users case:

- ENGSET FORMULA:

$$\Pi_{block} = \frac{A_i^C \binom{M-1}{C}}{\sum_{k=0}^C A_i^k \binom{M-1}{k}}$$

$$A_i = \frac{A_o}{M}$$

- Erlang-B can be re-obtained as limit case
 - $M \rightarrow \text{infinity}$
 - $A_i \rightarrow 0$
 - $M \cdot A_i \rightarrow A_o$
- Erlang-B is a very good approximation as long as:
 - A/M small (e.g. < 0.2)
- In any case, Erlang-B is a conservative formula
 - yields higher blocking probability
 - Good feature for planning



Capacity planning

→ Target: support users with a given Grade Of Service (GOS)

⇒ GOS expressed in terms of upper-bound for the blocking probability

→ GOS example: subscribers should find a line available in the 99% of the cases, i.e. they should be blocked in no more than 1% of the attempts

→ Given:

→ C channels

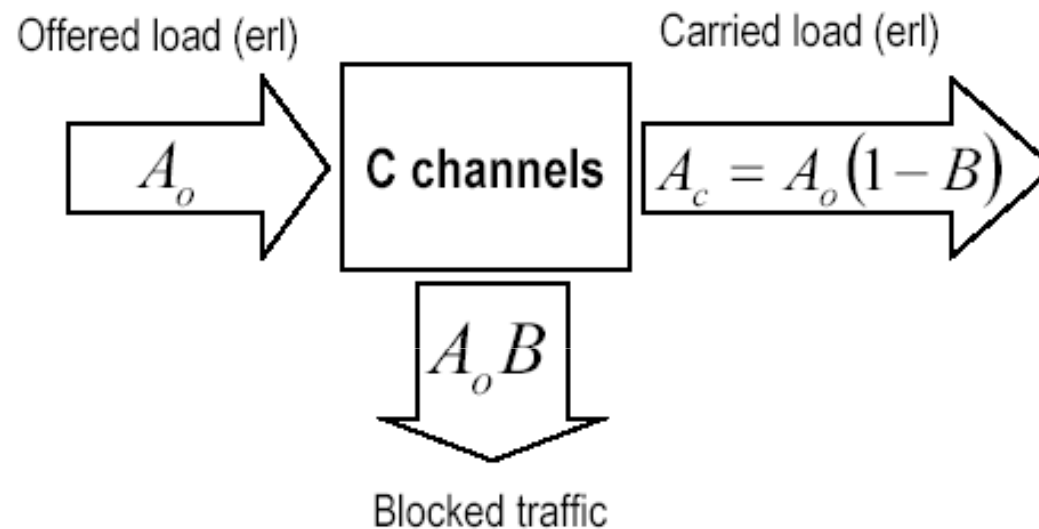
→ Offered load A_o

→ Target GOS B_{target}

⇒ C obtained from numerical inversion of $B_{\text{target}} = E_{1,C}(A_o)$



Channel usage efficiency

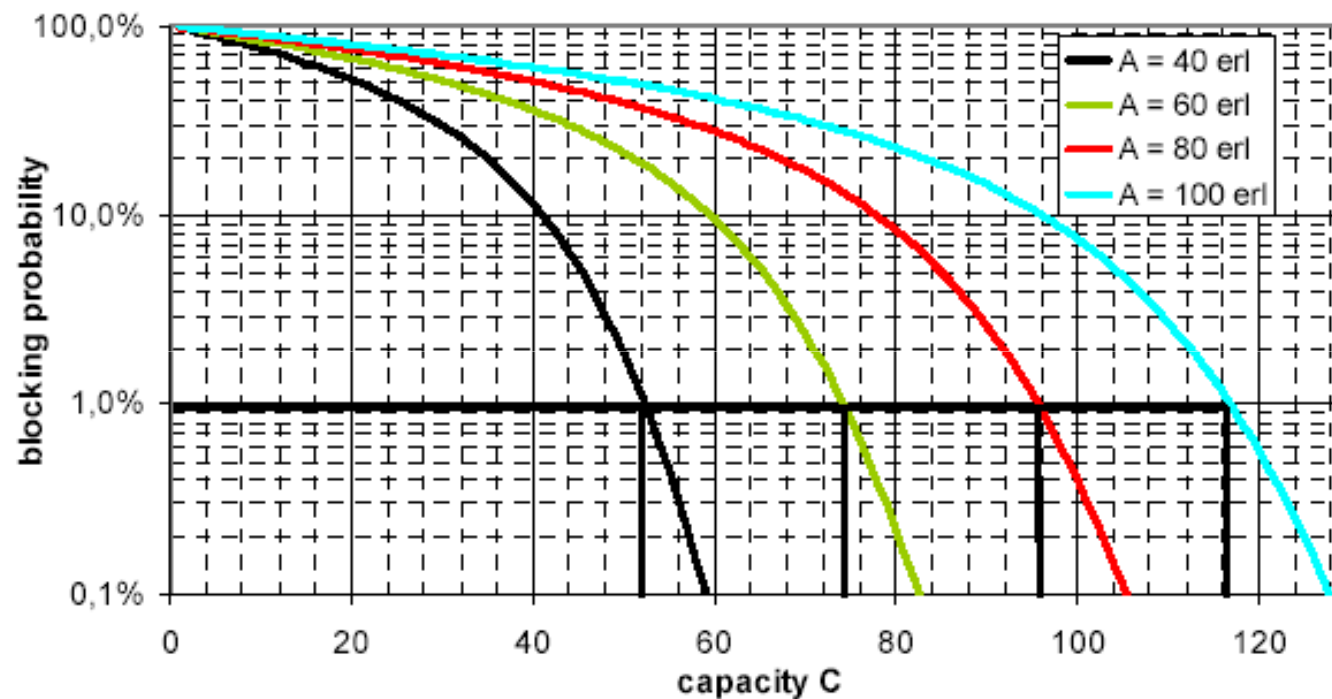


efficiency: $\eta = \frac{A_c}{C} = \frac{A_o(1 - E_{1,C}(A_o))}{C} \approx \frac{A_o}{C}$ if small blocking

Fundamental property: for same GOS, efficiency increases as C grows!!



example



GOS = 1% maximum blocking.
Resulting system dimensioning
and efficiency:

40 erl	$C \geq 53$	$\eta = 74.9\%$
60 erl	$C \geq 75$	$\eta = 79.3\%$
80 erl	$C \geq 96$	$\eta = 82.6\%$
100 erl	$C \geq 117$	$\eta = 84.6\%$



Application to cellular networks

**Cell size (radius R) may be determined
on the basis of traffic considerations**

→ First step:

- ⇒ Given num channels and GOS
 - C=50 available channels in a cell
 - Blocking probability ≤ 2%
- ⇒ Evaluate maximum cell (offered) load
 - From Erlang-B inversion (tables)
 - A=40.25 erl

→ Second step

- ⇒ Given traffic generated by each user
 - Each user: 4 calls/busy-hour
 - Each call: 2 min in average
 - $A_1 = 4 \times 2 / 60 = 0.1333$ erl/user
- ⇒ Evaluate max num of users in cell
 - M=301.87 ~ 302

→ Third step:

- ⇒ Given density of users
 - $\delta = 500$ users/km²
- ⇒ Evaluate cell radius

$$\delta = \frac{M}{\pi R^2} \Rightarrow R = \sqrt{\frac{M}{\pi \delta}}$$

- ⇒ R ~ 438m



Other example

→ Three service providers are planning to provide cellular service for an urban area. The target GOS is 2% blocking. Users make 3 calls/busy-hour, each lasting 3 minutes in average ($A_i = 3/20 = 0.15$)

⇒ **Question: how many users can support each provider?**

→ Provider A configuration: 20 cells, each with 40 channels

→ Provider B configuration: 30 cells, each with 30 channels

→ Provider C configuration: 40 cells, each with 20 channels

→ Provider A:

⇒ 40 channels/cell

⇒ at 2%: $A_0 = 30.99$ erl/cell

⇒ 619.8 erl-total

⇒ $M = 4132$ overall users

→ Provider B:

⇒ 30 channels/cell

⇒ at 2%: $A_0 = 21.93$ erl/cell

⇒ 654.9 erl-total

⇒ $M = 4386$ overall users

→ Provider C:

⇒ 20 channels/cell

⇒ at 2%: $A_0 = 13.18$ erl/cell

⇒ 527.2 erl-total

⇒ $M = 3515$ overall users

Compare case A with C! The reason is the lower efficiency of 20 channels versus 40



Sectorization and traffic

Co-Channel Interference (CCI)

- Assume cluster $K=7$
- Omnidirectional antennas: **CCI=18.7 dB**
- 120° sectors: **CCI=23.4 dB**
- 60° sectors: **CCI=26.4 dB**

- Sectorization yields to better CCI
- BUT: the price to pay is a much lower trunking efficiency!

→ With 60 channels/cell, **GOS=1%**,

⇒ Omni:	60 channels	$A_o = 1 \times 46.95 = 46.95$ erl	$\eta = 77.5\%$
⇒ 120°:	$60/3 = 20$ channels	$A_o = 3 \times 12.03 = 36.09$ erl	$\eta = 59.5\%$
⇒ 60°:	$60/6 = 10$ channels	$A_o = 6 \times 4.46 = 26.76$ erl	$\eta = 44.1\%$



conclusion

- ➔ This section has given some hints regarding:
 - ⇒ Cell planning via propagation considerations
 - ⇒ Cell planning via teletraffic consideration
- ➔ elementary models
 - ⇒ But sufficient to understand what's inside planning
- ➔ No mobility!
 - ⇒ Teletraffic models need to be extended to manage handover rates!
 - ⇒ Blocking requirement for an handover call **MUST** be much lower than blocking for a new incoming call
 - severe math complications
 - Guard channels for handover
 - Out of the scopes of this class!